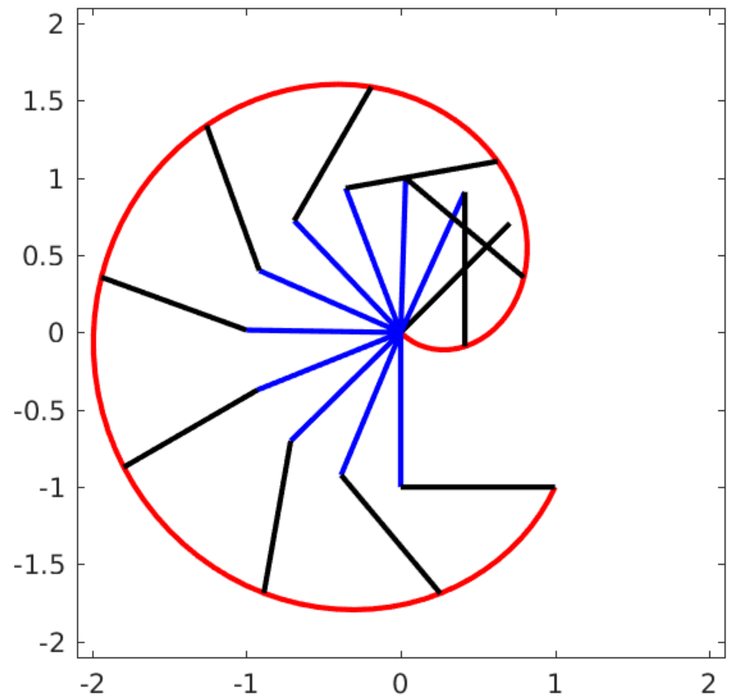
Day 12

Paths satisfying end point constraints

Joint-Space Path

- a joint-space path is computed considering the joint variables



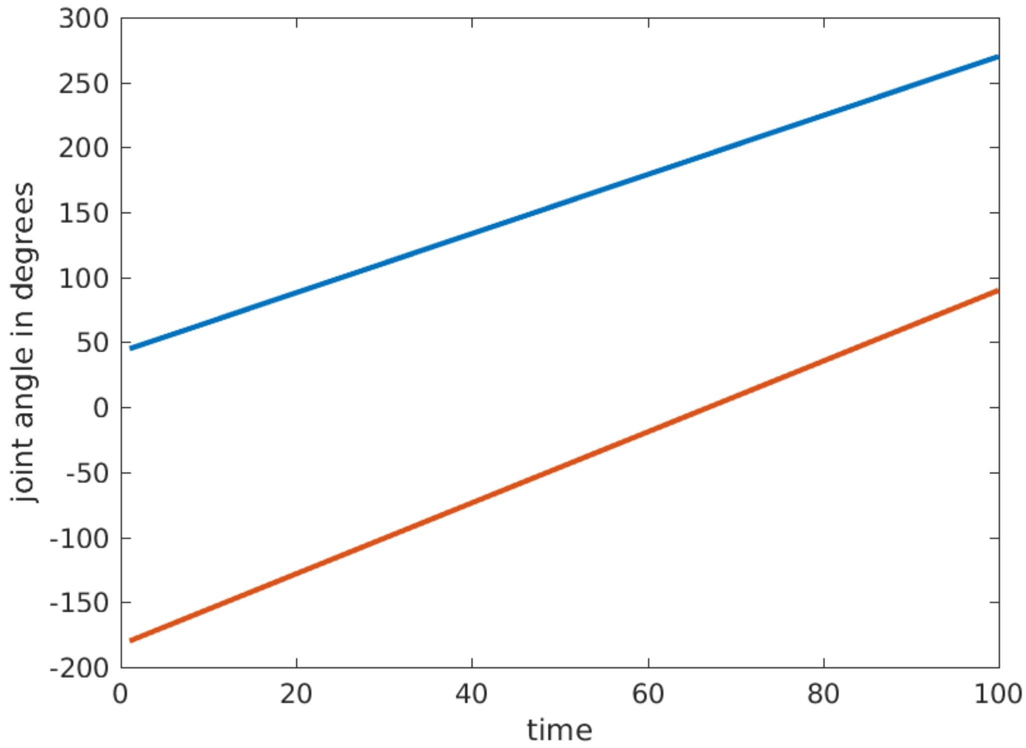
link 1

link 2

end effector path

Joint-Space Path Joint Angles

- linear joint-space path



Constraints

- in the previous example we had two constraints for joint 1:
  1. ${}^{0}\theta_1 = 60$
  2. ${}^{f}\theta_1 = 270$
- the simplest path satisfying these constraints is the straight line path
- if we add more constraints then a straight line path may not be able to satisfy all of the constraints

Velocity constraints

- a common constraint is that the robot starts from a stationary position and stops at a stationary positions
  - in other words, the joint velocities are zero at the start and end of the movement

3. $$^{0}\left(\frac{d\theta_1}{dt}\right) = {}^{0}\dot{\theta}_1 = 0$$
4. $$^{f}\left(\frac{d\theta_1}{dt}\right) = {}^{f}\dot{\theta}_1 = 0$$

- more generally, we might require non-zero velocities

3. $$^{0}\left(\frac{d\theta_1}{dt}\right) = {}^{0}\dot{\theta}_1 = {}^{0}v$$
4. $$^{f}\left(\frac{d\theta_1}{dt}\right) = {}^{f}\dot{\theta}_1 = {}^{f}v$$

Acceleration constraints

- for smooth motion, we might require that the acceleration at the start and end of the motion be zero

5. $$^{0}\left(\frac{d^2\theta_1}{dt^2}\right) = {}^{0}\ddot{\theta}_1 = 0$$

6. $$^{f}\left(\frac{d^2\theta_1}{dt^2}\right) = {}^{f}\ddot{\theta}_1 = 0$$

- more generally, we might require non-zero accelerations

5. $$^{0}\left(\frac{d^2\theta_1}{dt^2}\right) = {}^{0}\ddot{\theta}_1 = {}^{0}\alpha$$

6. $$^{f}\left(\frac{d^2\theta_1}{dt^2}\right) = {}^{f}\ddot{\theta}_1 = {}^{f}\alpha$$

Satisfying the constraints

- given some set of constraints on a joint variable q our goal is to find $q(t)$ that satisfies the constraints
- there are an infinite number of choices for $q(t)$
  - it is common to choose "simple" functions to represent $q(t)$

Satisfying the constraints with polynomials

- suppose that we choose $q(t)$ to be a polynomial
- if we have n constraints then we require a polynomial with n coefficients that can be chosen to satisfy the constraints
  - in other words, we require a polynomial of degree $(n-1)$

Satisfying the constraints with polynomials

- suppose that we have joint value and joint velocity constraints
  1. $q(t_0) = q_0$
  2. $q(t_f) = q_f$
  3. $\dot{q}(t_0) = v_0$
  4. $\dot{q}(t_f) = v_f$

- we require a polynomial of degree 3 to represent $q(t)$
  - $q(t) = a + bt + ct^2 + dt^3$

- the derivative of $q(t)$ is easy to compute
  - $\dot{q}(t) = b + 2ct + 3dt^2$

Satisfying the constraints with polynomials

- equating $q(t)$ and $\dot{q}(t)$ to each of the constraints yields:

1. $q(t_0) = q_0 = a + bt_0 + ct_0^2 + dt_0^3$
2. $q(t_f) = q_f = a + bt_f + ct_f^2 + dt_f^3$
3. $\dot{q}(t_0) = v_0 = b + 2ct_0 + 3dt_0^2$
4. $\dot{q}(t_f) = v_f = b + 2ct_f + 3dt_f^2$

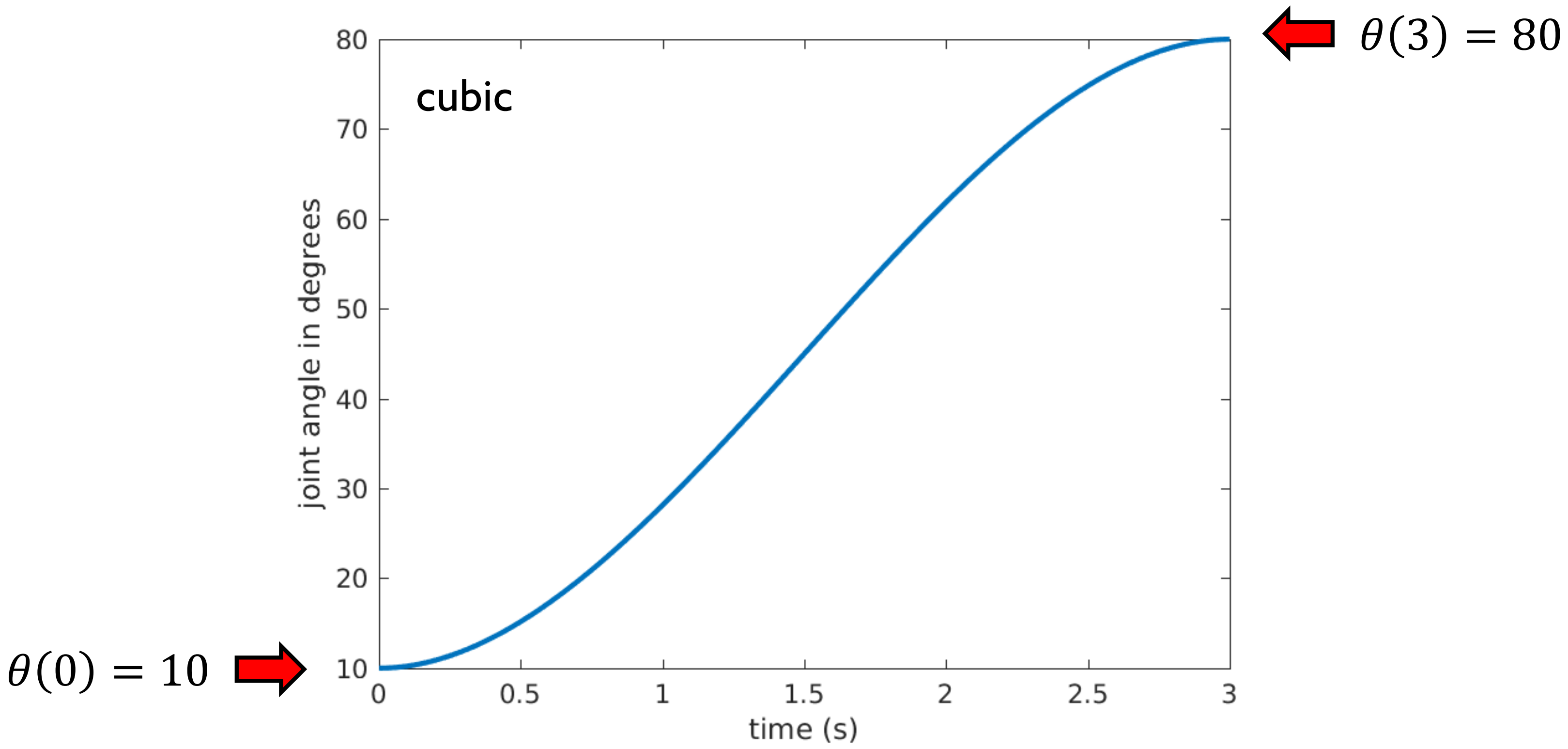
which is a linear system of 4 equations with 4 unknowns (a, b, c, d)

Example

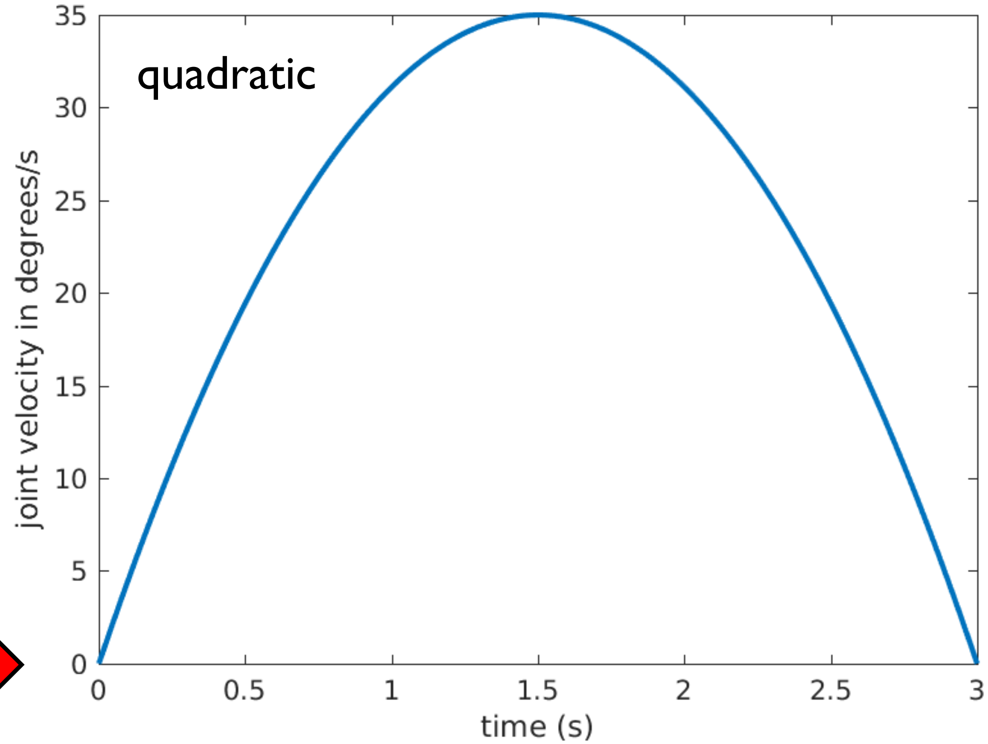
- consider the following constraints where the robot is stationary at the start and end of the movement

1. $q(t_0) = \theta(0) = 10$
2. $q(t_f) = \theta(3) = 80$
3. $\dot{q}(t_0) = \dot{\theta}(0) = 0$
4. $\dot{q}(t_f) = \dot{\theta}(3) = 0$

Example: Joint angle



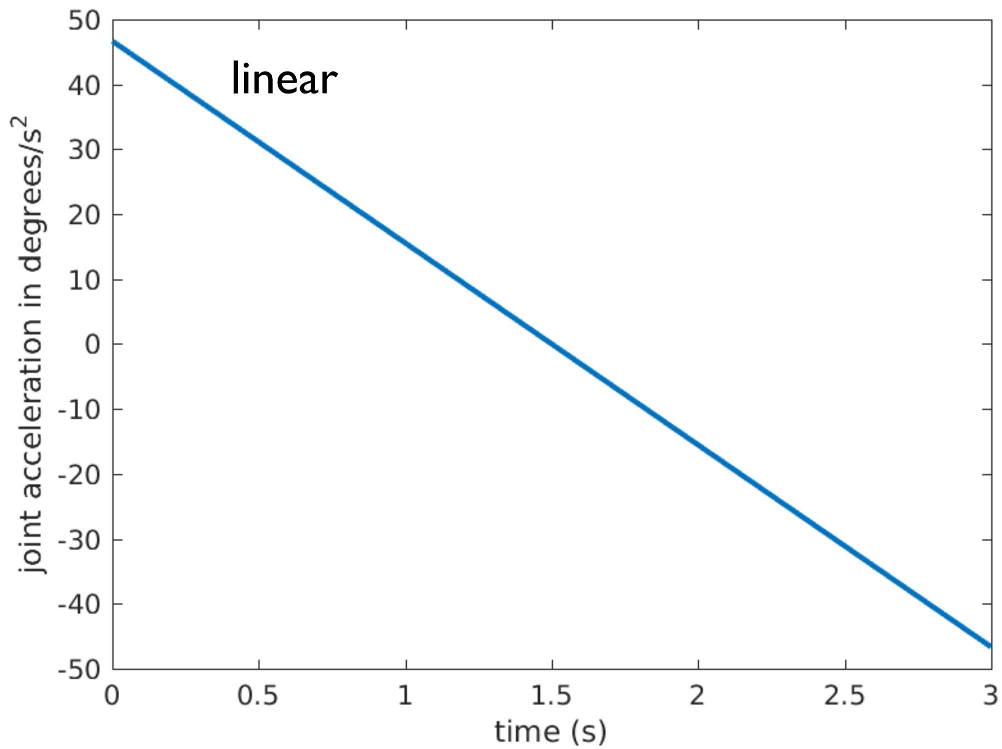
Example: Joint velocity



$\dot{\theta}(0) = 0$

$\dot{\theta}(3) = 0$

Example: Joint acceleration



Satisfying the constraints with polynomials

- suppose that we have joint value, joint velocity, and joint acceleration constraints
  1. $q(t_0) = q_0$
  2. $q(t_f) = q_f$
  3. $\dot{q}(t_0) = v_0$
  4. $\dot{q}(t_f) = v_f$
  5. $\ddot{q}(t_0) = \alpha_0$
  6. $\ddot{q}(t_f) = \alpha_f$

Satisfying the constraints with polynomials

- we require a polynomial of degree 5 to represent $q(t)$
  - $q(t) = a + bt + ct^2 + dt^3 + et^4 + ft^5$

- the derivatives of $q(t)$ are easy to compute
  - $\dot{q}(t) = b + 2ct + 3dt^2 + 4et^3 + 5ft^4$
  - $\ddot{q}(t) = 2c + 6dt + 12et^2 + 20ft^3$

Satisfying the constraints with polynomials

- equating $q(t), \dot{q}(t)$, and $\ddot{q}(t)$ to each of the constraints yields:

1. $q(t_0) = q_0 = a + bt_0 + ct_0^2 + dt_0^3$
2. $q(t_f) = q_f = a + bt_f + ct_f^2 + dt_f^3$
3. $\dot{q}(t_0) = v_0 = b + 2ct_0 + 3dt_0^2$
4. $\dot{q}(t_f) = v_f = b + 2ct_f + 3dt_f^2$
5. $\ddot{q}(t_0) = \alpha_0 = 2c + 6dt_0 + 12et_0^2 + 20ft_0^3$
6. $\ddot{q}(t_f) = \alpha_f = 2c + 6dt_f + 12et_f^2 + 20ft_f^3$

which is a linear system of 6 equations with 6 unknowns (a, b, c, d, e, f)

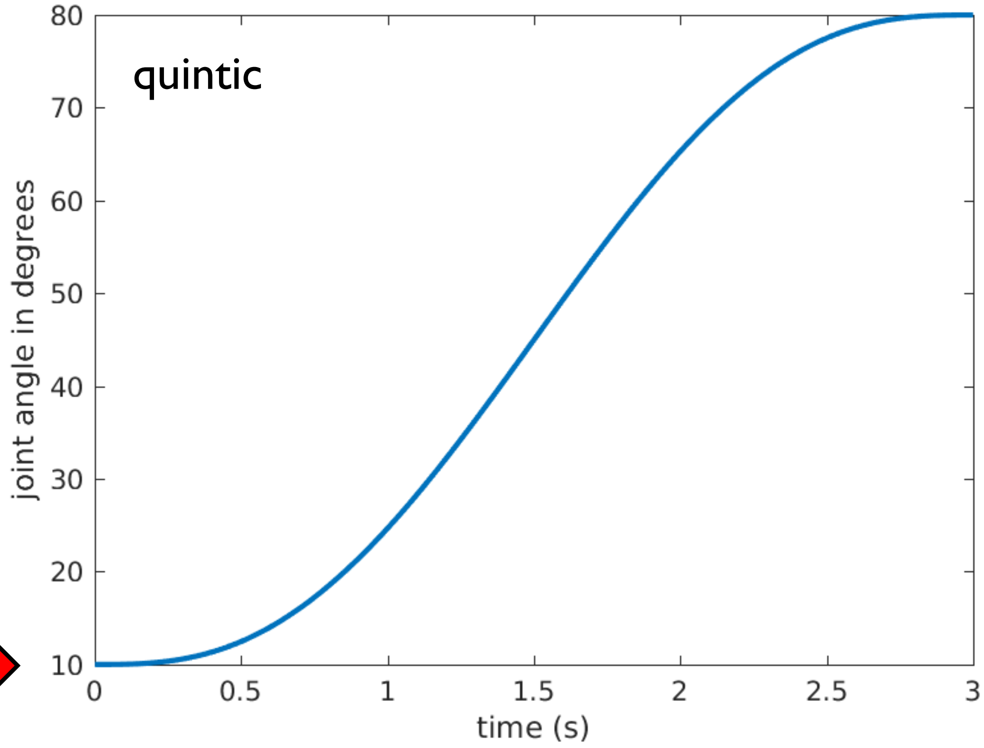
Example

- consider the following constraints where the robot is stationary at the start and end of the movement, and the joint accelerations are zero at the start and end of the movement

1. $q(t_0) = \theta(0) = 10$
2. $q(t_f) = \theta(3) = 80$
3. $\dot{q}(t_0) = \dot{\theta}(0) = 0$
4. $\dot{q}(t_f) = \dot{\theta}(3) = 0$
5. $\ddot{q}(t_0) = \ddot{\theta}(0) = 0$
6. $\ddot{q}(t_f) = \ddot{\theta}(3) = 0$

Example: Joint angle

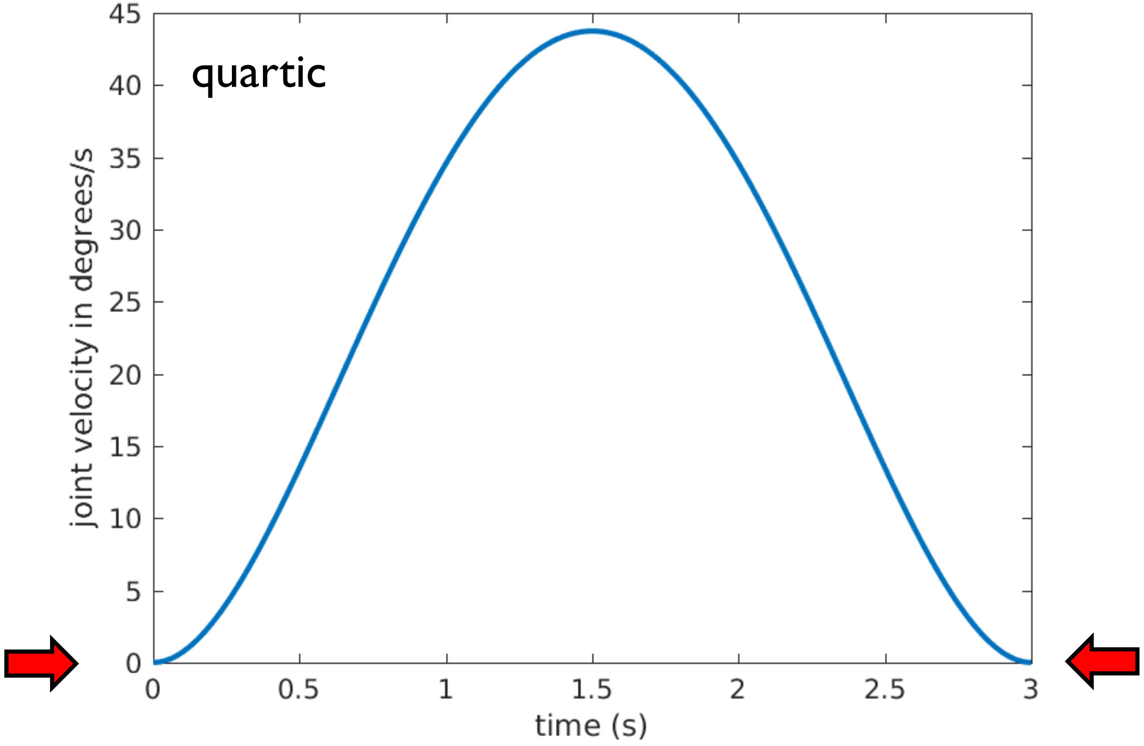
$\theta(3) = 80$



$\theta(0) = 10$

Example: Joint velocity

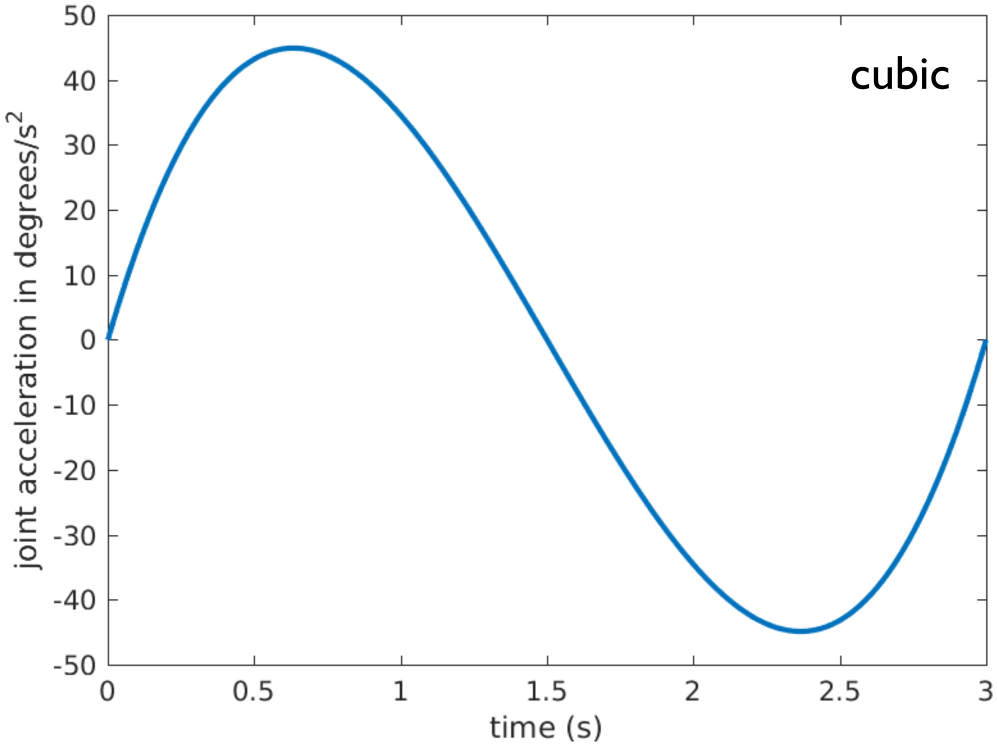
$\dot{\theta}(0) = 0$



$\dot{\theta}(3) = 0$

Example: Joint acceleration

$\ddot{\theta}(0) = 0$



$\ddot{\theta}(3) = 0$